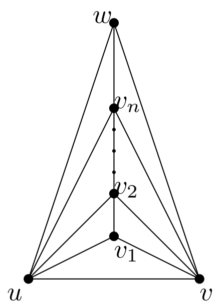
Exercise sheet 4

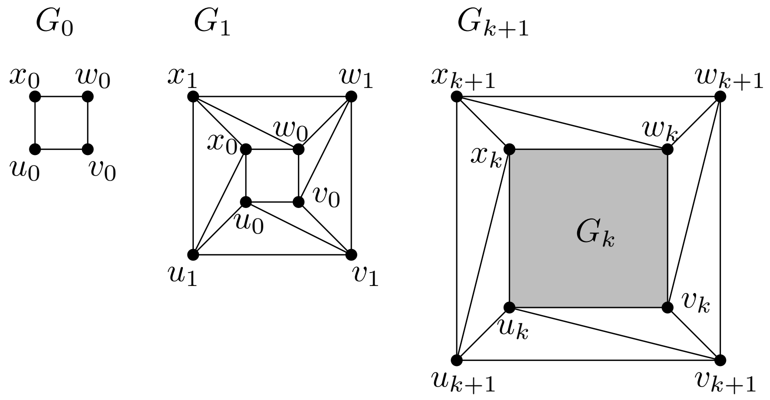
Exercise 1 – Barycentric coordinates - exponential area

Let G be the graph on $n+3$ vertices shown in the figure below. The outerface of G consists of vertices u, v, w. The remaining vertices induce a path $v_1, \ldots, v_n$ In the interior of G, such that v_i is adjacent to vertices u and v, $i = 1, \ldots, n$, and v_n is also adjacent to w. Prove that the planar straight-line drawing of G cumputed using barycentric coordinates, requires exponential area. Recall that the barycentric coordinates (x_v, y_v) of an interior vertex v are given by $x_v = \frac{1}{d_v} \sum_{u \in N(v)} x_u$ and $y_v = \frac{1}{d_v} \sum_{u \in N(v)} y_u$, where $N(v)$ is the set of neighbors of v and $d_v = |N(v)|$. **4 Points**



Exercise 2 – Planar graphs with quadratic area

Let G_k be the graph on $4k$ vertices defined recursively as follows (see following figure). For $k = 0$, G_0 consists of a 4-cycle u_0, v_0, w_0, x_0. Let u_k, v_k, w_k, x_k be the vertices of G_k on its outerface. Graph G_{k+1} is constructed from G_k by adding a 4-cycle $u_{k+1}, v_{k+1}, w_{k+1}, x_{k+1}$ in the outerface of G_k, and edges $u_{k+1}u_k$, $u_{k+1}x_k$, $v_{k+1}v_k$, $v_{k+1}u_k$, $w_{k+1}w_k$, $w_{k+1}v_k$, $x_{k+1}x_k$ and $x_{k+1}w_k$ as shown below.



Prove that any straight-line grid drawing of G_k requires width and height linear in k. **4 Points**

Exercise 3 – Canonical Order of outerplanar graphs

A graph is outerplanar if it has a planar embedding such that all vertices are on the same face, usually the outer face. It is a maximal outerplanar graph if it is internally triangulated. Describe a special canonical order built precisely for maximal outerplanar graphs.

a) Reformulate the conditions (C1)-(C3) for maximal outerplanar graphs. Can we enforce a bound on the degree of v_{k+1}? **2 Points**

b) How can we use the algorithm for maximal planar graphs to obtain a canonical order for maximal outerplanar graphs? **4 Points**

Due by: Thursday, December 8 by 6pm.
