Department of Mathematics
National Technical University of Athens

Graph Drawing
12 December 2022

Exercise sheet 5

Exercise 1 – Higher degree vertices in orthogonal layouts

Let $G = (V, E)$ be an arbitrary graph with an embedding $\mathcal{E}$. Our goal is to draw G orthogonally, while preserving the embedding, such that all vertices of degree greater than 4 are represented by rectangles instead of points. To achieve this, we replace every vertex v having $\deg(v) > 4$ by a ring of vertices $v_1, \ldots, v_{\deg(v)}$, such that the edges incident to v are distributed among the vertices $v_1, \ldots, v_{\deg v}$ (see figure below). The embedding $\mathcal{E}$ is modified accordingly during this step. Let G' with embedding $\mathcal{E}'$ be the result of this replacement step.

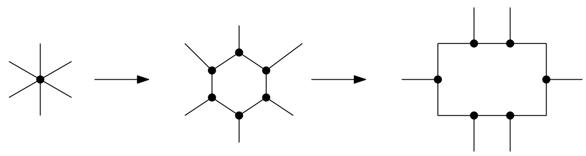

Figure 1: Replacement of a vertex of higher degree by a ring, which shall be represented by a rectangle in the orthogonal drawing.

Modify the flow network by Tamassia such that it provides a bend-minimal orthogonal description of $(G', \mathcal{E}')$ in which every ring representing a vertex of higher degree is a rectangle such that no vertices are placed in any of its four corners and such that every side of the rectangle contains at least one vertex. **7 Points**

Hint: Consider the set V' of the new vertices and the set E' of the new edges of a ring that are added to the graph after the modification. Think of the additional constraints to the flow model to enforce the structure of the ring and its vertices.

Exercise 2 – Existence of minimum cost flow

Let $G = (V, E)$ be a plane graph with a given embedding, and let $N(G)$ be the constructed flow network (as described in the lecture). Recall that any valid flow X of minimum cost k corresponds to an orthogonal description $H(G)$ of G with the minimum number of bends (i.e. k bends). Prove that X exists, that is, there exists at least one valid flow for $N(G)$.

4 Points

Hint: Construct a valid flow algorithmically.

Exercise 3 – Edge bending left and right in orthogonal representation

Let $G = (V, E)$ be a planar graph with embedding $\mathcal{E}$ and let $H(G)$ be a bend-minimal orthogonal description of $(G, \mathcal{E})$. Is it possible that there exists an edge such that, in $H(G)$, it bends to the right as well as to the left?

Prove this claim (by giving an example) or disprove it (by showing that such an edge cannot exist). **3 Points**

Due by: Thursday, December 22 by 6pm.
