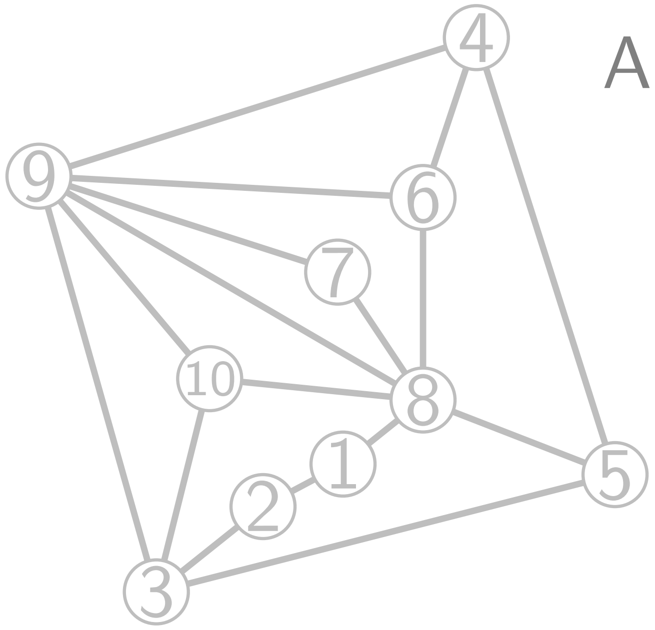
Visualisation of graphs

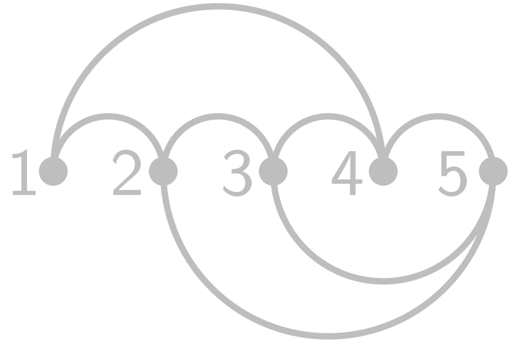
Introduction

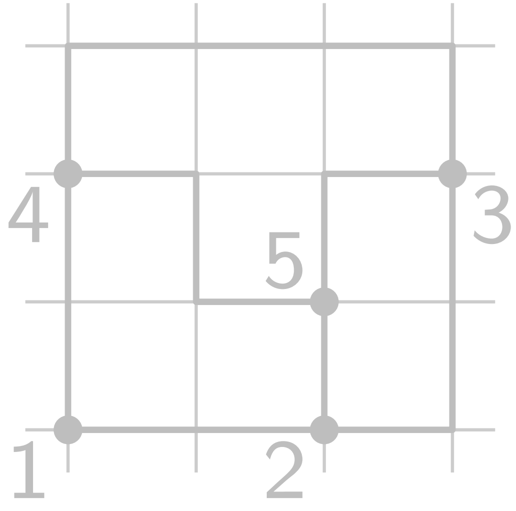
The graph visualisation problem

Antonios Symvonis · Chrysanthi Raftopoulou

Fall semester 2022







The slides of this presentation were created by researchers at Karlsruhe Institute of Technology (KIT), TU Wien, U Wuerzburg, U Konstanz, ...

Graphs and their representations

What is a graph?

- graph $G = (V, E)$
- vertices $V = \{v_1, v_2, \ldots, v_n\}$
- edge $E = \{e_1, e_2, \ldots, e_m\}$

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- Set notation

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- Adjacency list

v_1: v_2, v_8
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v_4: v_5, v_6, v_9
v_5: v_3, v_4, v_8
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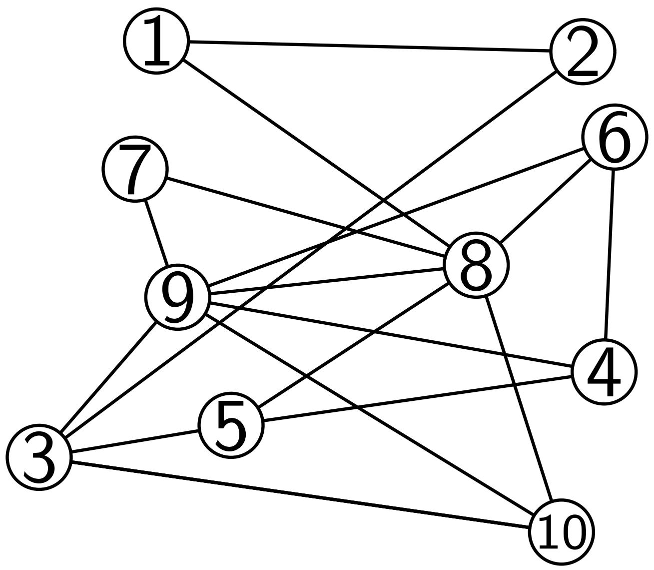
- Drawing



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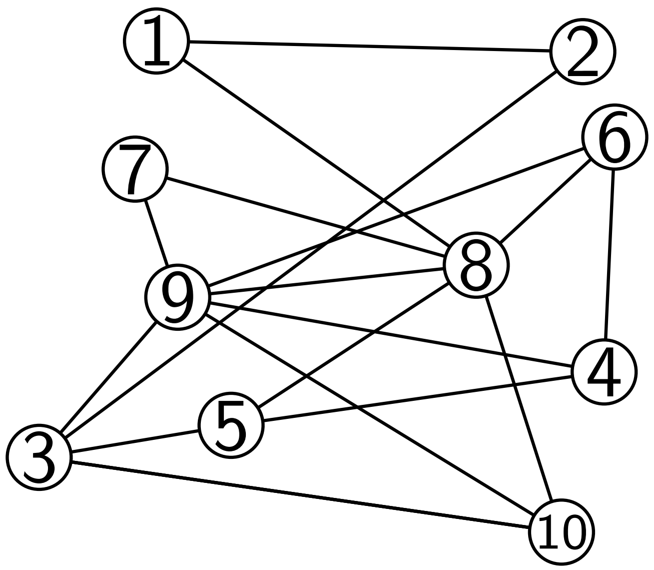

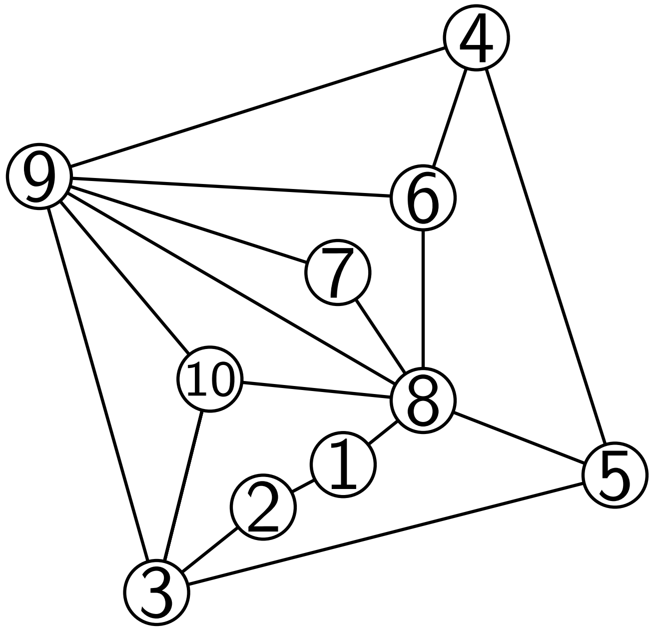

Why draw graphs?

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Graphs are a mathematical representation of real physical and abstract networks.

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Abstract networks

- Social networks
- Communication networks
- Phylogenetic networks
- Metabolic networks
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Physical networks

- Metro systems
- Road networks
- Power grids
- Telecommunication networks
- Integrated circuits
- . . .

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We need algorithms that draw graphs automatically to make networks more accessible to humans.

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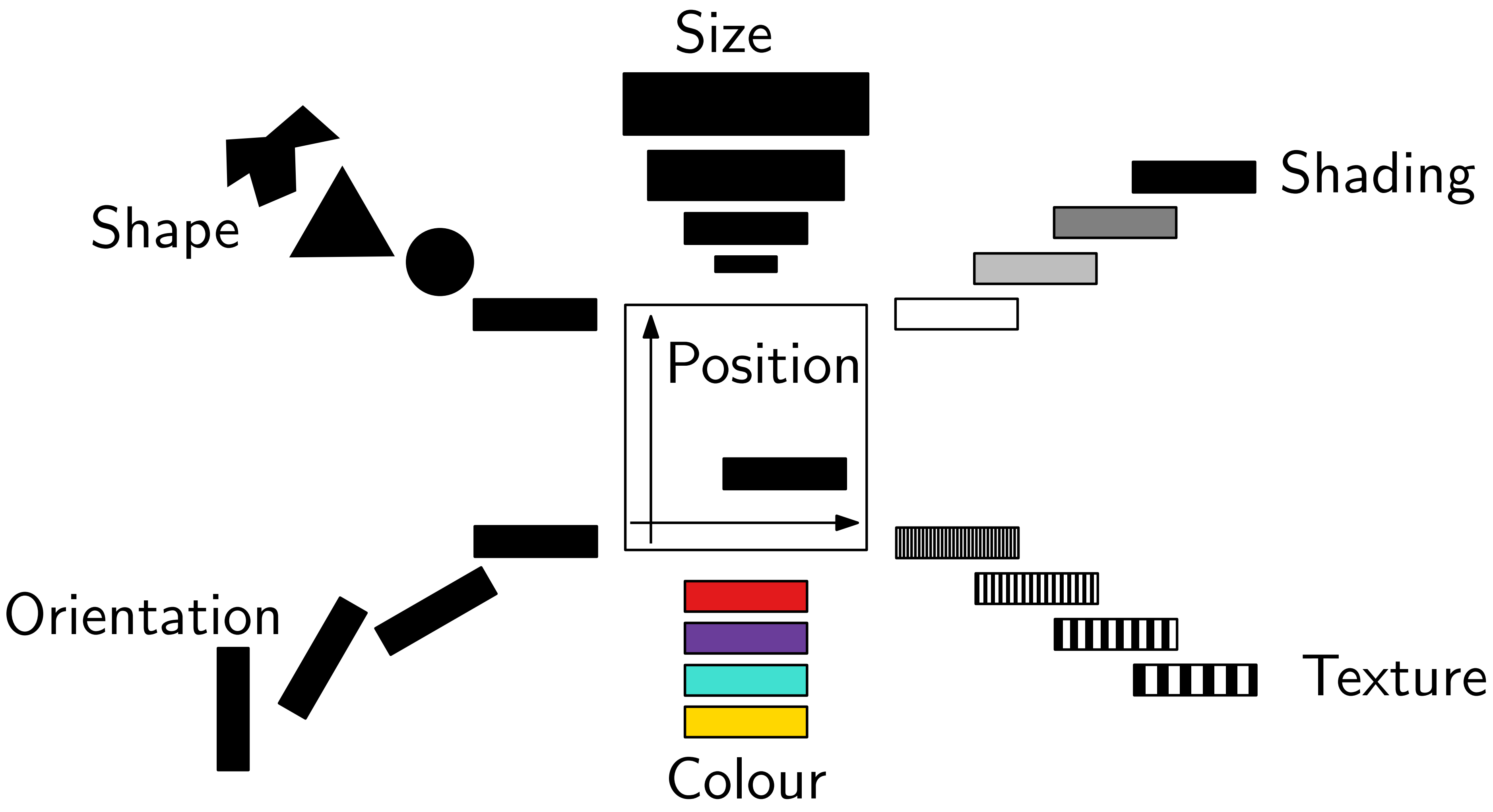

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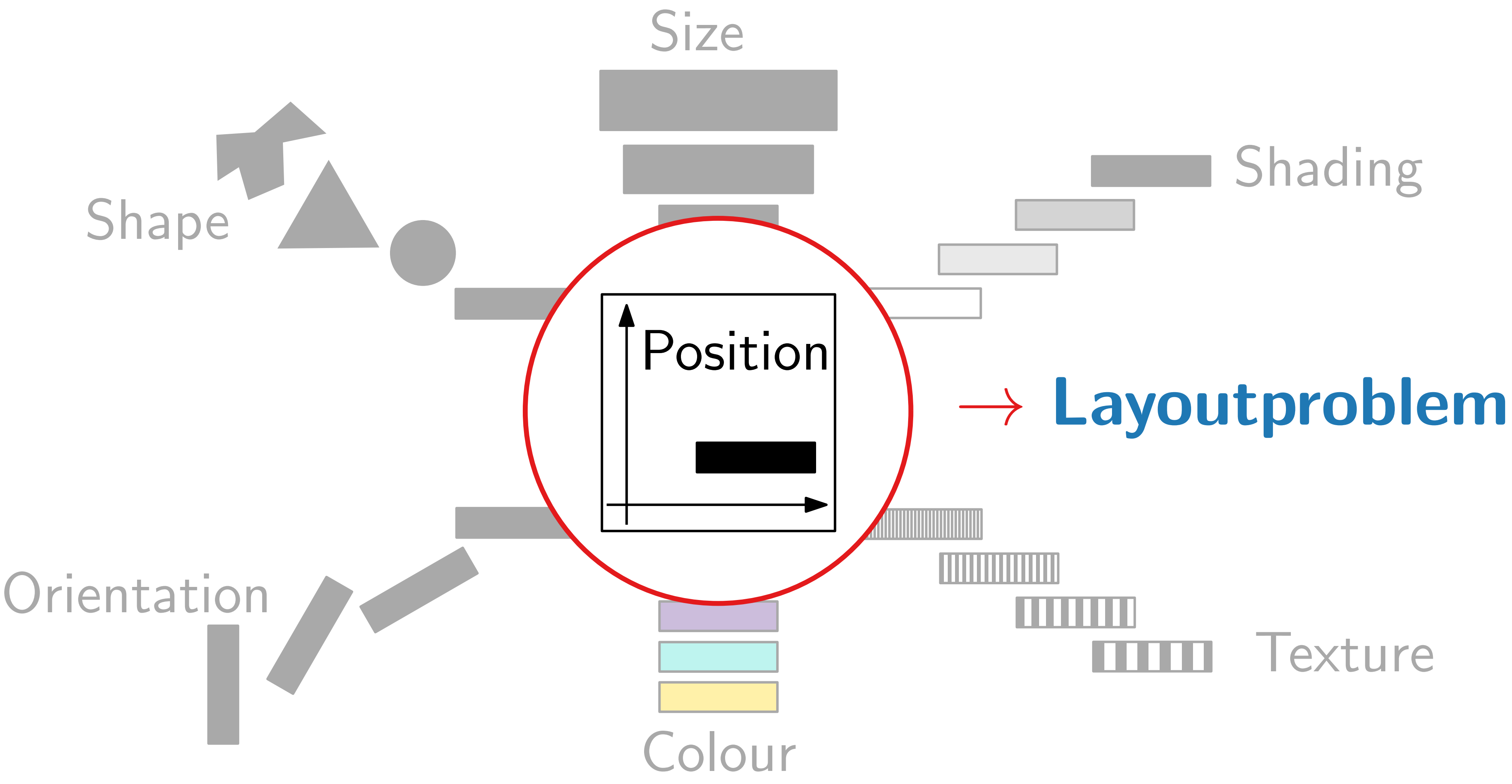

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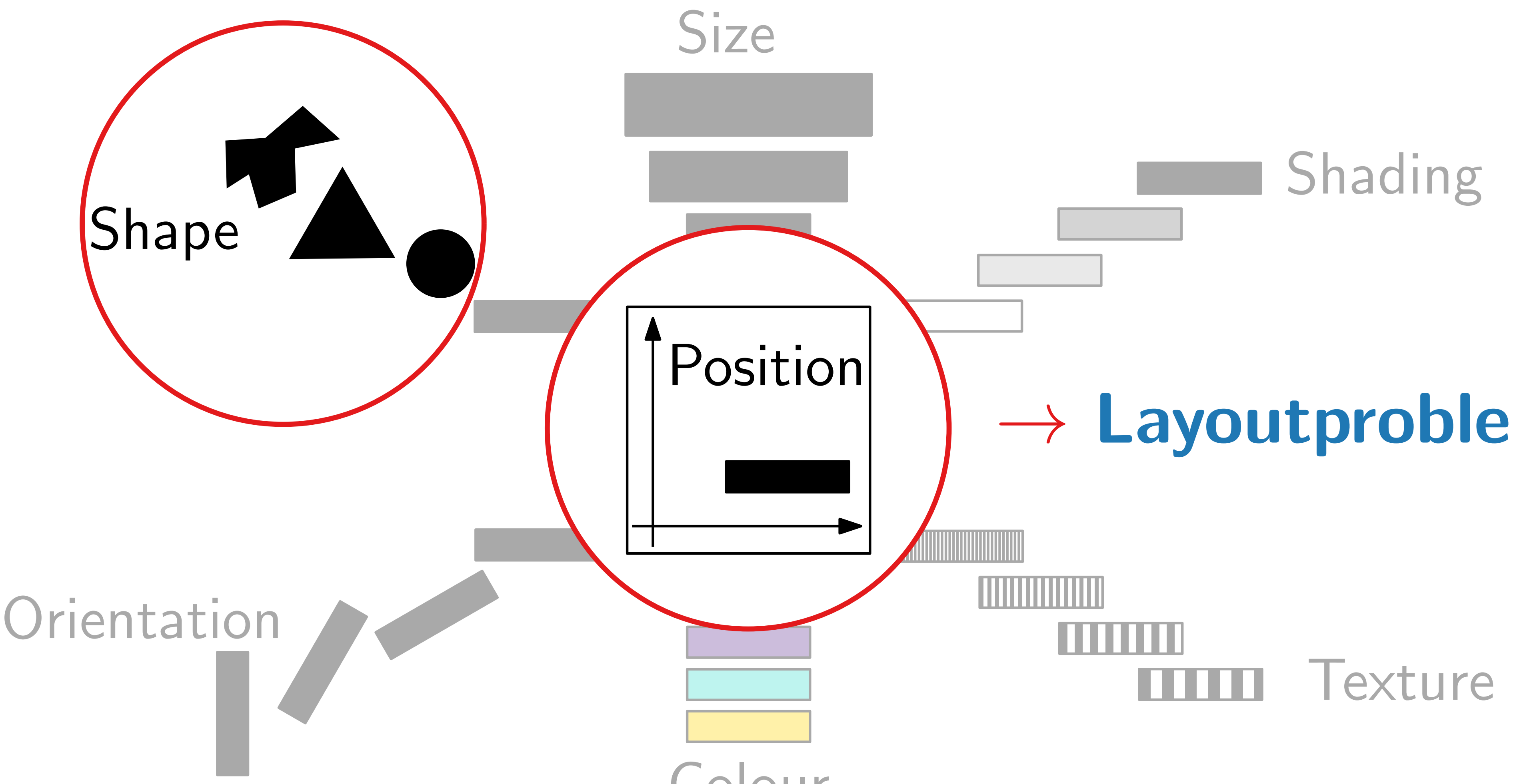
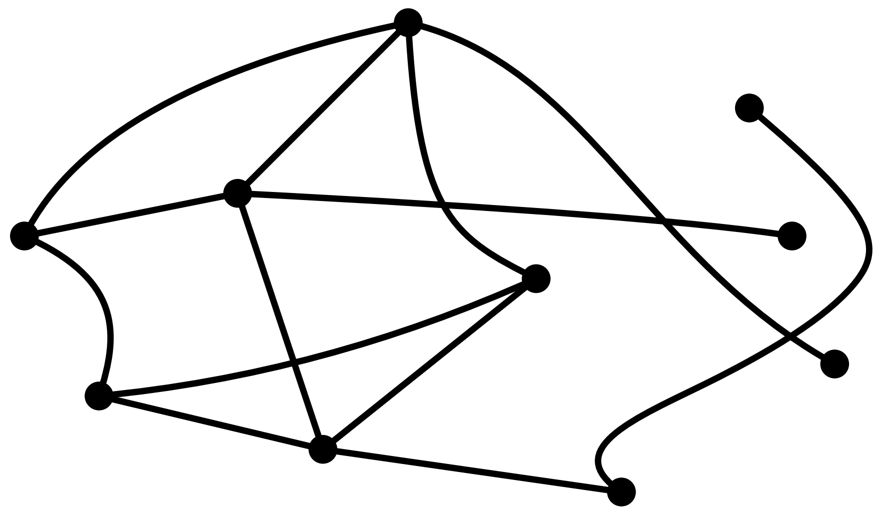


→ **Layoutproblem**

The layout problem

- *Here* restricted to the **standard representation**, so-called node-link diagrams.



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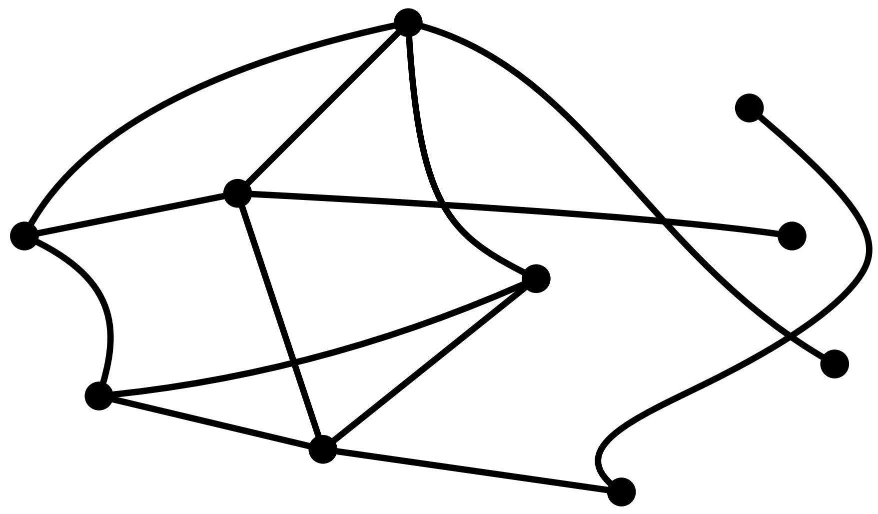

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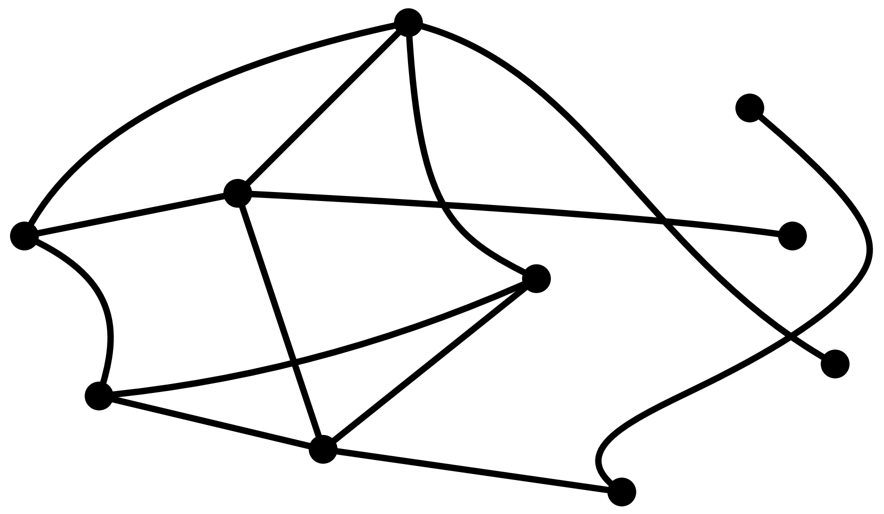

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in: Graph $G = (V, E)$

out: **nice** drawing Γ of G

- $\Gamma \colon V \to \mathbb{R}^2$, vertex $v \mapsto$ point $\Gamma(v)$
- $\Gamma \colon E \to$ curves in $\mathbb{R}^2$, edge $\{u, v\} \mapsto$ simple, open curve $\Gamma(\{u, v\})$ with endpoints $\Gamma(u)$ und $\Gamma(v)$

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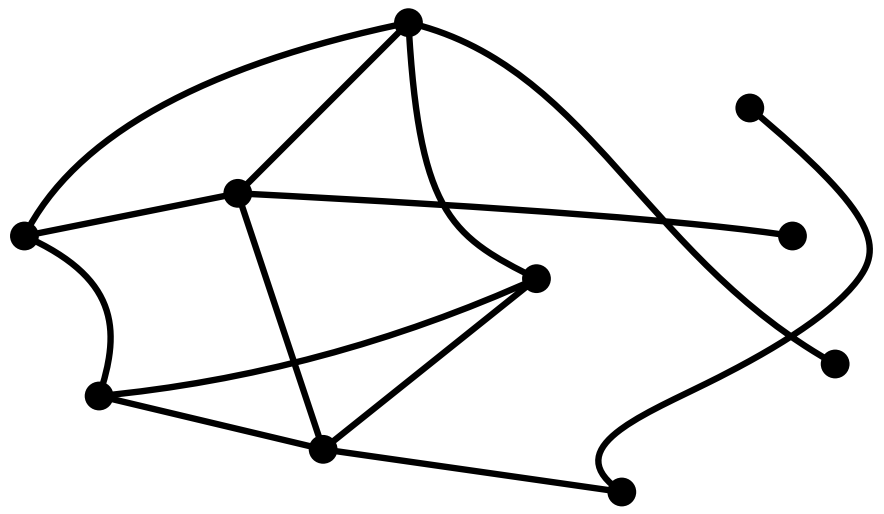

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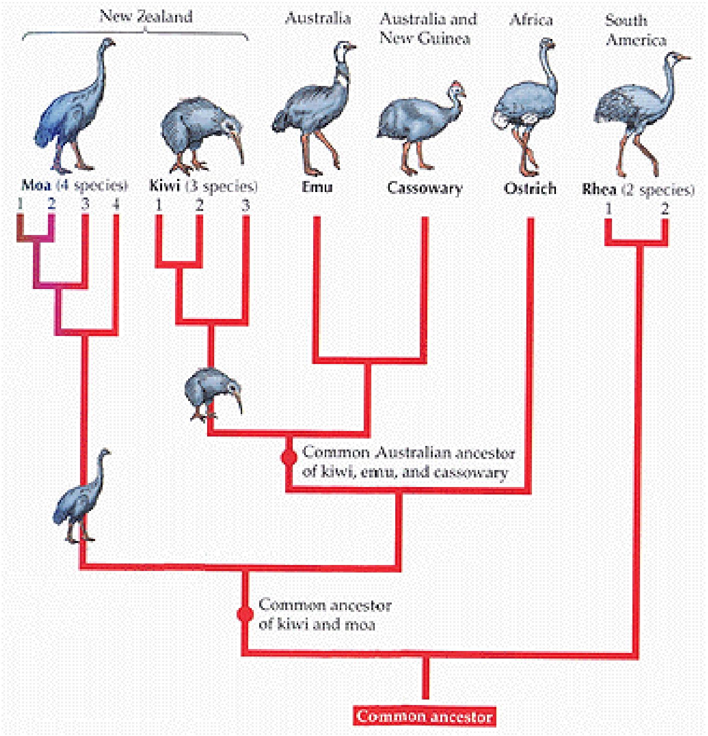
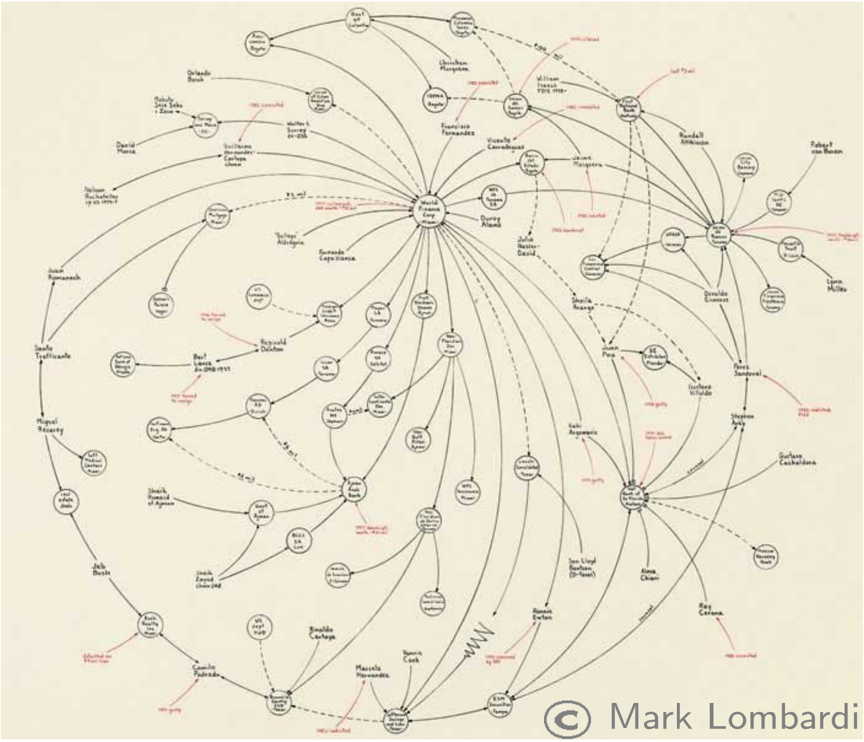
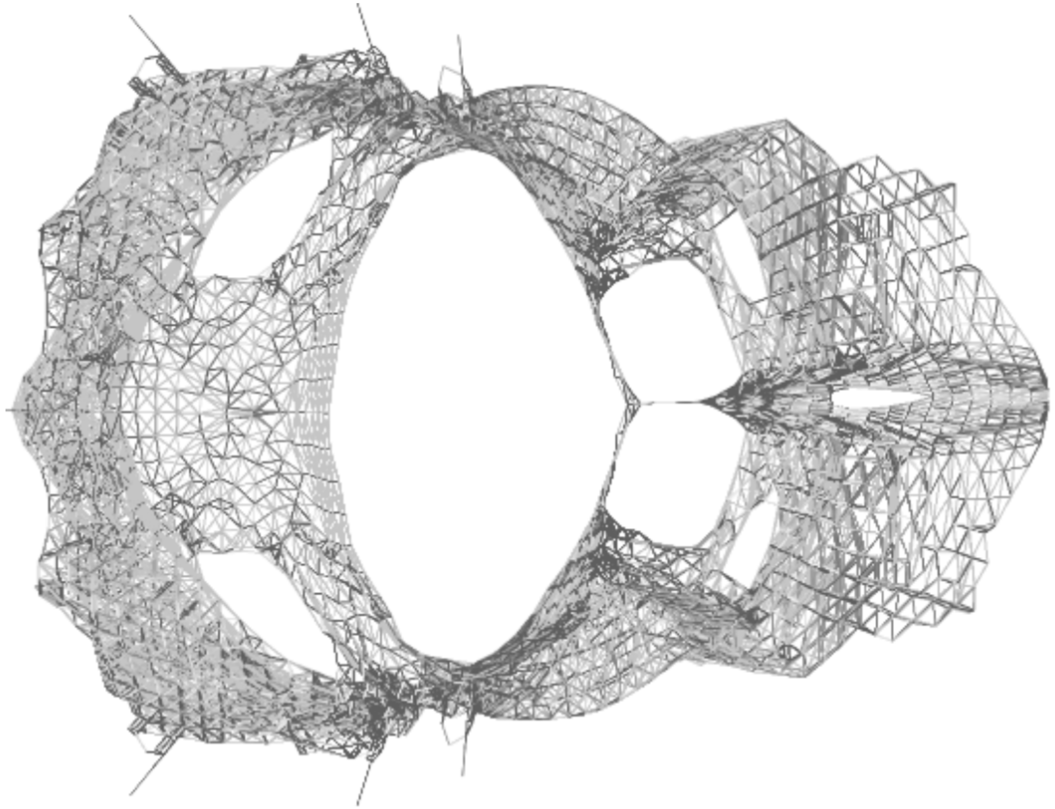
But what is a **nice** drawing?

Examples







- See slides with more examples.

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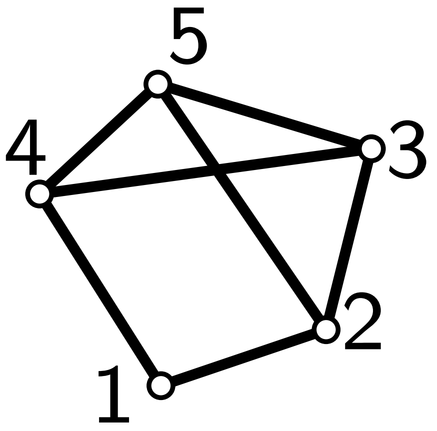

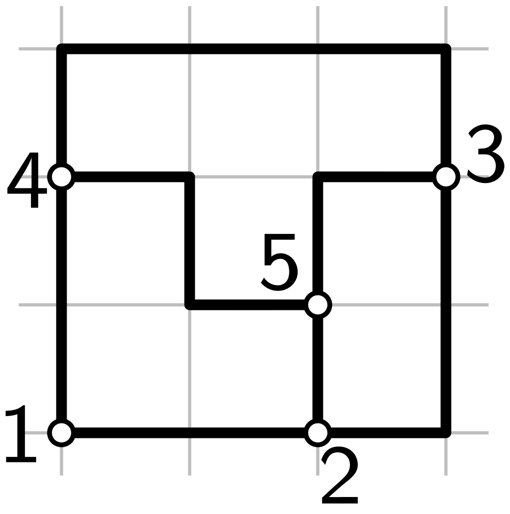

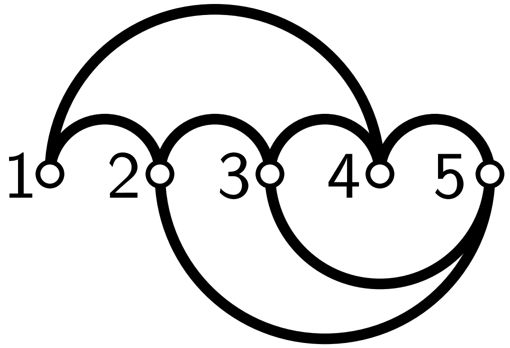

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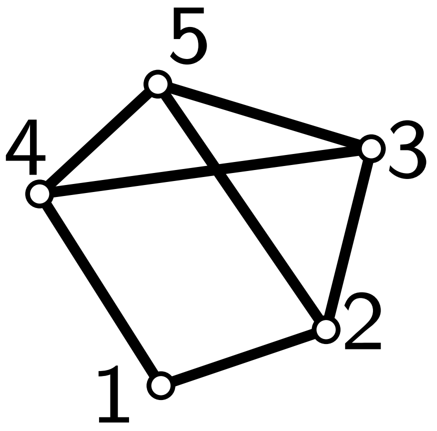

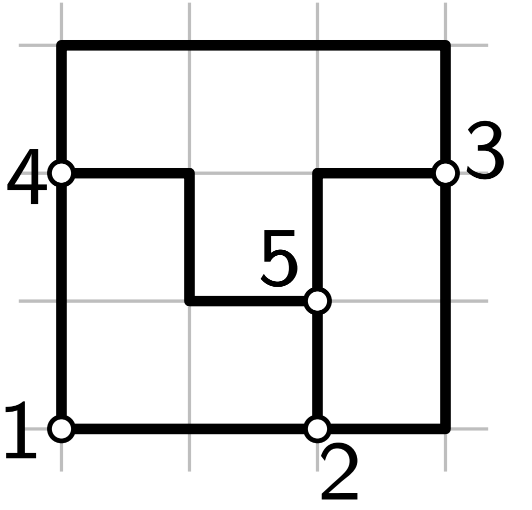

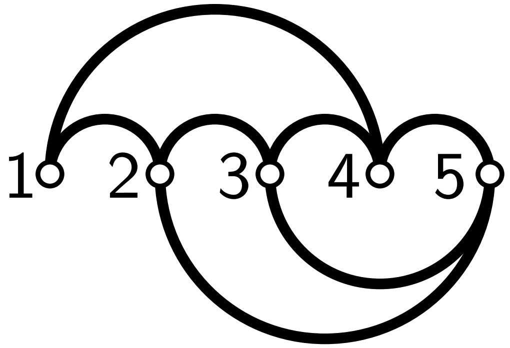

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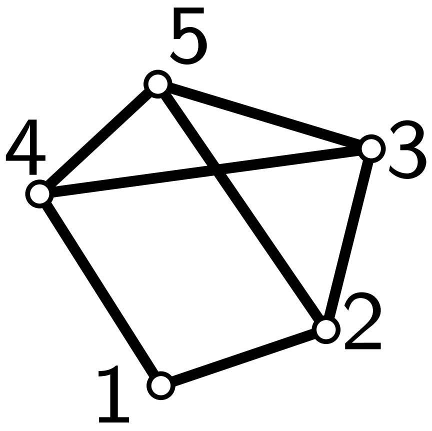

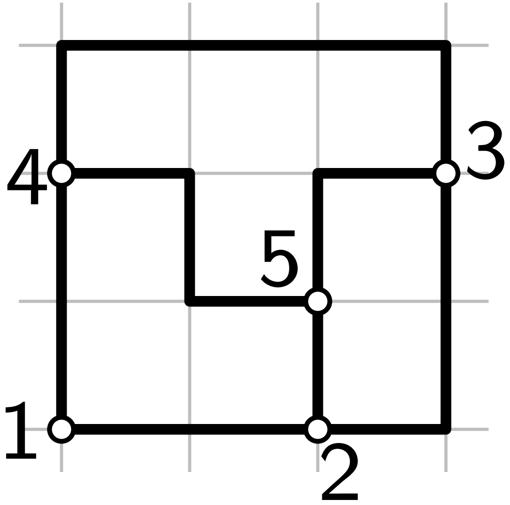

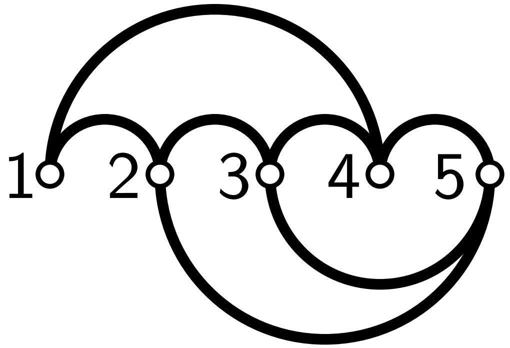

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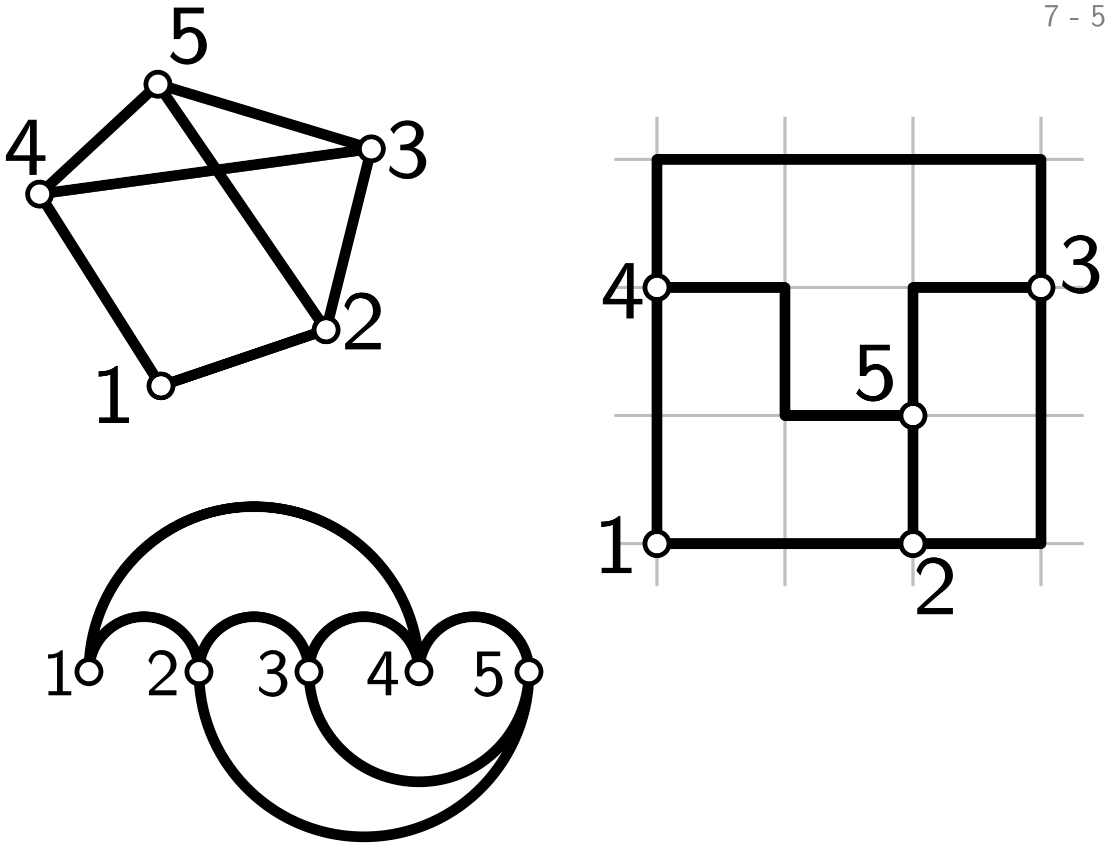

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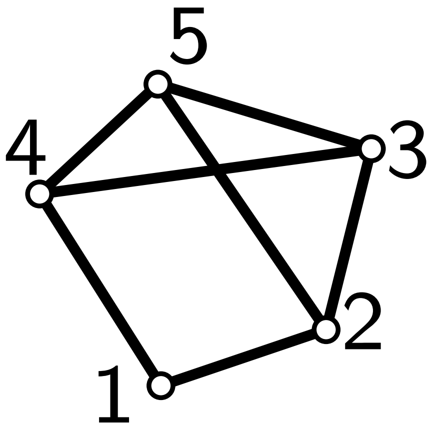

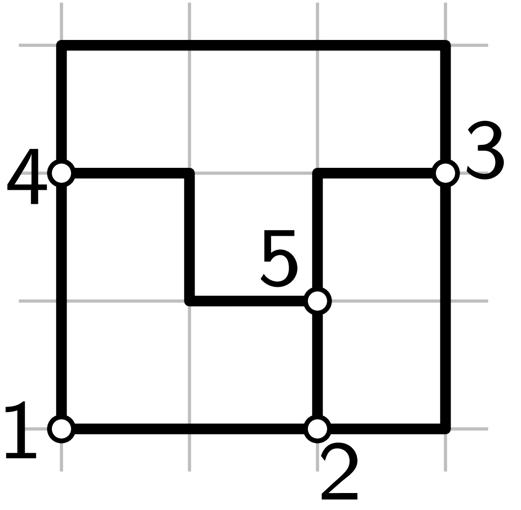

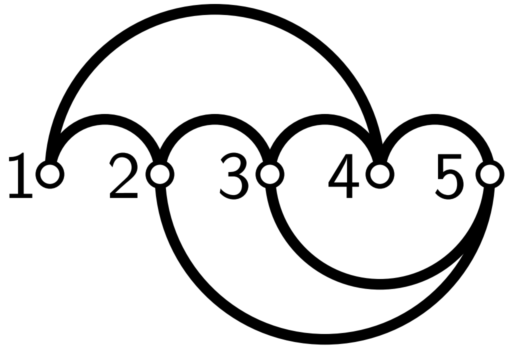

→ lead to NP-hard optimization problems
→ such criteria are often inversely related

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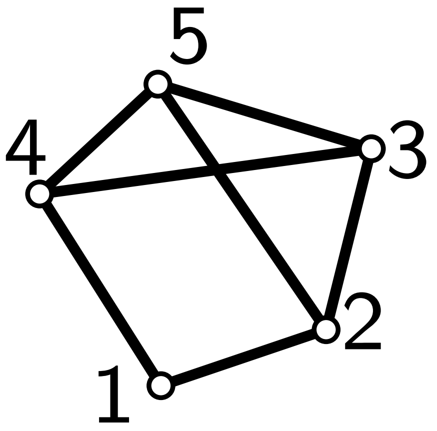
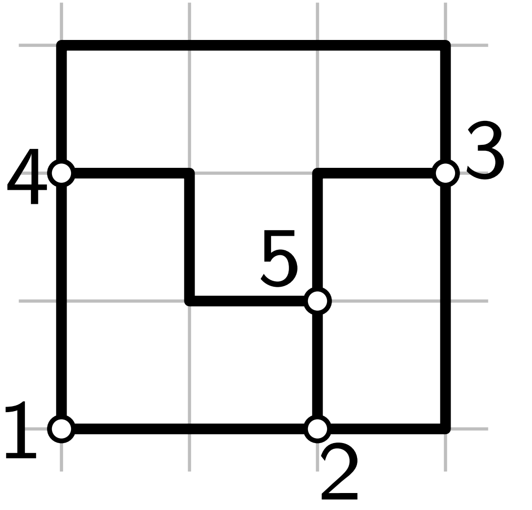
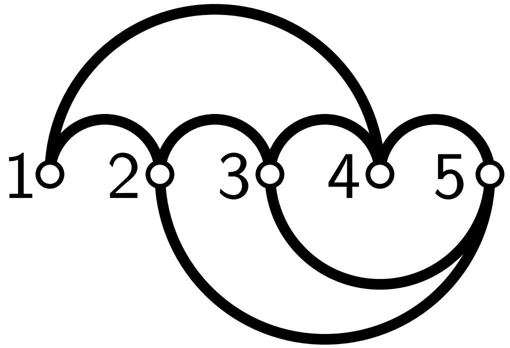
3. Local Constraints, e.g.



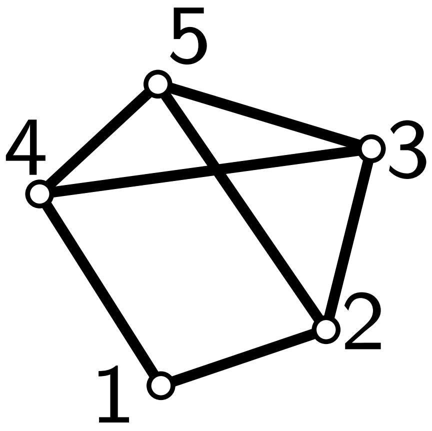


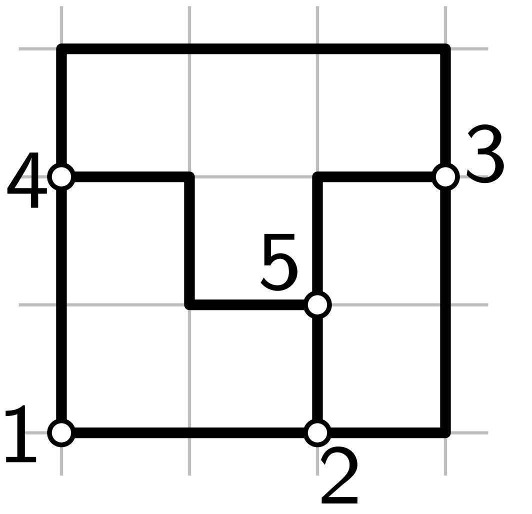


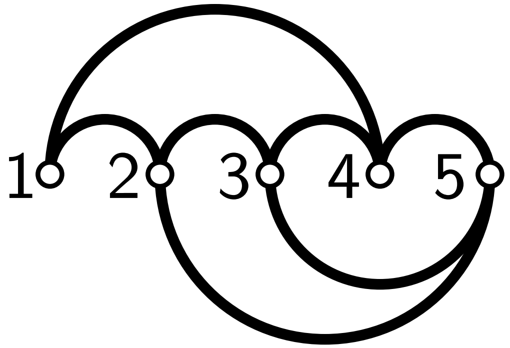
→ lead to NP-hard optimization problems

→ such criteria are often inversely related

Requirements of a graph layout







1. Drawing conventions and requirements, e.g.,
   - straight edges with $\Gamma(uv) = \overline{\Gamma(u)\Gamma(v)}$
   - orthogonal edges (i.e. with bends)
   - grid drawings
   - without crossing
2. Aesthetics to be optimised, e.g.
   - crossing/bend minimisation
   - edge length uniformity
   - minimising total edge length/drawing area
   - angular resolution
   - symmetry/structure

→ lead to NP-hard optimization problems
→ such criteria are often inversely related

3. Local Constraints, e.g.
   - restrictions on neighbouring vertices (e.g., “upward”).
   - restrictions on groups of vertices/edges (e.g., “clustered”).

The layout problem

Graph visualisation problem

in: Graph $G = (V, E)$

out: Drawing Γ of G such that

The layout problem

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- **drawing conventions** are met,

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The layout problem

Graph visualisation problem

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- **drawing conventions** are met,
- **aesthetic criteria** are optimised, and
- some **additional constraints** are satisfied.

- Many algorithmically interesting questions arise.
- Rendering problem downstream is ignored.